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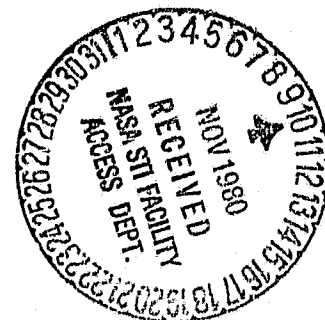
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The Minimum Flux Corona - Theory or Concept?

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Summary

The reply by Hearn (1979) to the criticisms of the minimum flux theory by Antiochos and Underwood (1978) are discussed. It is shown that these criticisms are correct in substance, as well as in detail.

Introduction

Following the statement of Hearn (1975) that his minimum flux corona theory gives results of interest to stellar coronae in general, not to just those which may be around OB supergiants and Wolf Rayet Stars, and to the solar corona in particular (Hearn 1977) it has been widely used as a tool for the investigation of stellar coronae (see e.g. the review by Mewe, 1979). On the other hand, many authors have criticized various aspects of the concept. (Antiochos and Underwood, 1978; Vaiana and Rosner, 1978; Endler et al., 1979; Van Tend, 1979; Mangeney and Souffrin, 1979). Hearn (1979) has replied to some of these criticisms, in particular to ours (Antiochos and Underwood, loc. cit., hereafter referred to as AU) and those of Vaiana and Rosner. The purpose of this letter is to provide counter-arguments to support our assertion that the "minimum flux corona theory" is untenable, because of inconsistencies and errors in its formulation.

Since studies of stellar coronae must use, as a basis, what is known about the solar corona, it is as well to begin with a recapitulation of the present state of knowledge.

Heating of the Upper Solar Atmosphere and Coronal Length Scales

Although the exact mechanisms by which the upper layers of the solar atmosphere are heated are not known, the general principles governing the energy balance of the chromosphere, transition region and corona are well understood. It is believed that mechanical energy originating in the photospheric and sub-photospheric layers propagate upwards as waves (e.g. acoustic or MHD waves) or quasi-static motions (twisting or displacement of magnetic flux tubes) to be dissipated in the higher layers. The spatial distribution of energy deposition will depend strongly on the kind of wave in question and the mode of its dissipation.

pation. If this is known, or some assumptions about it can be made, then in the absence of energy losses by the solar wind, the profile of temperature, density, conductive flux, etc. in the atmosphere may be found by solving, subject to appropriate boundary conditions, the heat equation

$$\nabla \cdot F_c = \epsilon - E_R \quad (1)$$

Here F_c is the conducted heat flux, ϵ the energy input and E_R the radiative losses per cubic centimeter (the plasma is assumed to be optically thin). Note that in using equation (1) we are implicitly assuming that the corona is static. In addition, it is necessary to specify the pressure distribution in the atmosphere, for example, by assuming hydrostatic equilibrium.

$$\frac{dp}{dz} = -\rho g \quad (2)$$

where z is the vertical coordinate and ρ the density.

Equation (1) may be solved in a one-dimensional form by assuming a plane parallel atmosphere stratified perpendicular to the z direction (Moore and Fung, 1974), or a loop geometry, with the spatial coordinate taken along the loop (Vesecky, et al., 1979). The latter authors assumed a spatially constant energy input, as did Rosner et al. (1978). Alternatively, deposition of energy over a scale length L dependent on the form of the energy input mechanism may be assumed.

The resulting solutions of (1) will be self-consistent. The scale lengths for the variation of pressure, temperature and so on will emerge automatically and will be consistent with the gravitational scale height and the scale length L for energy deposition. (Endler et al., 1979 have correctly pointed out the

importance of L in determining coronal scale lengths.) Any additional assumptions regarding the thermal structure of the atmosphere over-constrain the problem. They will either be redundant or lead to inconsistencies.

The minimum flux corona theory leads to exactly such inconsistencies. For instance Hearn (1979) regards the coronal energy source as a vanishingly thin layer, sandwiched between the transition region and corona, "where mechanical dissipation is necessary to convert transition region temperature gradients into an almost isothermal corona". Given the boundary condition that the conductive flux F_c vanishes above the photosphere, the one-dimensional form of equation (1) could be solved for such a point energy source. With such an absence of mechanical dissipation within the corona, it would be heated by energy conducted upward from this source and have a temperature maximum at its base. However Hearn (1975) makes the a priori assumption that the corona is cooled by heat conducted downward. His solution of the energy equation (20), for the coronal region only, with no energy sources, amplifies this contradiction.

Our original criticism of the minimum flux corona theory (AU), was directed at another inconsistency, resulting from the idea that the pressure scale height and scale length for temperature variation ("thickness of the conduction zone") can be independently and arbitrarily chosen. Although Hearn nowhere discusses length scales, this idea is implicit in his work. This discussion need not be repeated here. It should be noted, however, that this criticism is independent of the distribution of mechanical energy input. We did not, as Hearn suggests, neglect mechanical energy input. On the contrary, the inclusion of mechanical energy strengthens our argument because it is clear that the size scale for energy dissipation also cannot be neglected, although Hearn claims to have calculated the energy losses of all stellar coronae as a function of temperature and pressure only, independent of the coronal size scales.

Our point regarding the consistency of Hearn's analysis essentially ended with the sentence, "Thus the computation of F will be valid only for a small range of temperature near T_0 ". In the remainder of section 3, we showed that the heat equation, together with appropriate boundary conditions, can yield constraints on the fluxes which render an additional "minimum flux condition" superfluous and contradictory. For example, the boundary condition that the conducted flux vanish at the base of the transition region requires approximate equality of F_R and F_C , since most of the energy is radiated away in the high temperature region of the atmosphere, i.e. the corona. This assertion is discussed in detail by Vesecky et al. (1979) who support it with numerical examples using a realistic radiation loss curve. This same relation, the so-called "scaling law" for coronal loop has been found by other authors, e.g. Rosner et al. (1978); Craig et al. (1978); Hood and Priest (1979). Note that it is obtained by simply solving equations (1) and (2) for a particular set of boundary conditions and particular form for the heating function ϵ . It does not invoke any additional assumptions like a minimum flux hypothesis.

Stability

In his original paper Hearn (1975) computed F_R , F_C and F_W , the equivalent energy flux due to the stellar wind (see below) as a function of temperature T and pressure P . Finding that, for a given P , the curve of $F = F_R + F_C + F_W$ versus T passed through a minimum, he made the conjecture that a corona would be stable only if it occupied this position in the F - T diagram, with P such that energy losses equalled the energy input. However, he gave no justification for and certainly no proof of this statement.

In our criticism (AU) we invoked a thermal stability criterion (Field 1965) to argue that the unique stability of the "minimum flux point" could not be established without a proper perturbation analysis, as in Antiochos (1979). There

may well be other stable points in the diagram, or alternatively the "minimum flux point" may not be stable at all. Hearn (1979) now admits that he has no proof that the corona will sit at the minimum flux point, although through the use of a quantitative argument invoking time constants for the contraction of the corona and transition region, he has attempted to show that the thermal stability criterion does not apply to a corona. He concludes that either (a) every point on the F-T curve, for a given P is stable against perturbations in temperature or b) perturbations move a corona toward the minimum flux point. This contradicts his original conjecture, and reinforces our point that the uniqueness of the minimum flux point cannot be established without a proper, numerical, stability analysis.

Therefore, the objection to the minimum flux idea that we raised in our original criticism (AU) still remains, i.e. there is no reason to believe that the minimum flux point is the only stable point, as claimed by Hearn (1975). Since this claim is the whole basis for Hearn's model, then without a proof of this claim there is no reason to ascribe any validity to the minimum flux model.

The Calculation of F_W

Hearn (1975) calculates the energy balance of a corona from its base at radius r_0 out to the critical point, at radius r_c . To be in steady-state balance the energy input to this volume via the heating mechanism, conduction and mass flow must be equal to that carried out by conduction, radiation and mass flow. This condition, which is independent of what happens outside the volume (e.g. in the photosphere or at infinity) can be expressed by modifying (1) to

$$\nabla \cdot (\vec{F}_c + \vec{u}\epsilon + \vec{u}p) = \epsilon - E_R \quad (3)$$

where the new term $\vec{u}_e + \vec{u}_p$ represents the enthalpy flux; the symbols are defined in (AU).

Integrating over the coronal volume from r_o to r_c , we find that

$$\left[4\pi r^2 (F_c + u_e + u_p) \right]_{r_o}^{r_c} = 4\pi \int_{r_o}^{r_c} \epsilon r^2 dr - 4\pi \int_{r_o}^{r_c} E_R r^2 dr \quad (4)$$

The various terms in (4) can be converted to equivalent energy fluxes at the base of the corona by dividing by $4\pi r_o^2$. Thus the energy flux heating the corona is

$$F_{in} \equiv \frac{1}{r_o^2} \int_{r_o}^{r_c} \epsilon r^2 dr \quad (5)$$

and this must be equal to the sum of the energy flux due to radiative losses:

$$F_R \equiv \frac{1}{r_o^2} \int_{r_o}^{r_c} E_R r^2 dr, \quad (6)$$

to conductive losses:

$$F_c \equiv \left(\frac{r_c}{r_o} \right)^2 F_c(r_c) - F_c(r_o) \quad (7)$$

and to mass flow (solar/stellar wind):

$$F_W \equiv \left(\frac{r_c}{r_o} \right)^2 [u_e + u_p]_{r_c} - [u_e + u_p]_{r_o} \quad (8)$$

Hearn's (1975) expression for F_W (his equation 6) may be rewritten:

$$F_W \equiv \left(\frac{r_c}{r_o} \right)^2 [u_e]_{r_o} \quad (9)$$

From this comparison it may be seen that Hearn (1975) neglects a) the work \vec{u}_p

required for expansion and b) the energy carried into the base of the corona by the wind. Also, it is clear that in the equation $F_{in} = F_R + F_c + F_W$, equation (8) is the correct expression to use and not equation (9) as claimed by Hearn (1979).

Application to the Solar Corona and Coronal Holes

On the basis of the minimum flux corona theory, Hearn (1977) stated that "the main differences between a coronal hole and quiet coronal regions are explained by a reduction of the thermal conduction coefficient by transverse components of this magnetic field in the transition zone of quiet coronal regions". The required reduction factor is $\cos \theta$, where θ is the angle between the magnetic field direction and the vertical. Both (AU) and Vaiana and Rosner (1978) have pointed out that X-ray observations show the real difference between the quiet corona and coronal holes to be that the former is structured into loops by the magnetic field, whereas the field in coronal holes is open and, hence, permits energy loss into the solar wind. Hearn (1979) claims that his original suggestion is still valid under these conditions, since the structuring into loops is equivalent to lengthening of the conduction path by the loop structure of the magnetic field, which can be expressed by the $\cos \theta$ factor. This is evidently not so, for the suggestion cannot account for the difference between a loop structure and coronal hole region in which the magnetic field lines pass through the transition region at the same angle, e.g. $\theta = 0$.

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